





If λ_0 is an eigenvalue of an $n \times n$ matrix A , then the dimension of the eigenspace corresponding to λ_0 is called the **geometric multiplicity** of λ_0 , and the number of times that $\lambda - \lambda_0$ appears as a factor in the characteristic polynomial of A is called the **algebraic multiplicity** of λ_0 .

Theorem 5.2.4 Geometric and Algebraic Multiplicity

If A is a square matrix, then:

- a) For every eigenvalue of A , the geometric multiplicity is less than or equal to the algebraic multiplicity.
- b) A is diagonalizable if and only if its characteristic polynomial can be expressed as a product of linear factors, and the geometric multiplicity of every eigenvalue is equal to the algebraic multiplicity.

#14 Find the geometric and algebraic multiplicity of each eigenvalue of the matrix A , and determine whether A is diagonalizable. If A is diagonalizable, then find a matrix P that diagonalizes A , and find $P^{-1}AP$.

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$$

Theorem 5.2.2

- a) If $\lambda_1, \lambda_2, \dots, \lambda_k$ are distinct eigenvalues of a matrix A , and if $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ are corresponding eigenvectors, then $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$ is a linearly independent set.
- b) An $n \times n$ matrix with n distinct eigenvalues is diagonalizable.

Similarity

Definition: A transformation of the form $A \rightarrow P^{-1}AP$ is called a **similarity transformation**.

Definition: If A and B are square matrices, then we say that **B is similar to A** (or that A and B are **similar matrices**) if there is an invertible matrix P such that $B = P^{-1}AP$.

Example: Prove that if A and B are similar matrices, then $\det(A) = \det(B)$.

Any property that is preserved by a similarity transformation is called a **similarity invariant** and is said to be **invariant under similarity**.
